

DYNAMICS

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Introduction

- Relationship between force and motion
- Important in
 - Design of robots
 - Simulation and animation of robot motion
 - Design of control algorithm
- Euler-Lagrange Equation
- Several examples
- Several important properties

The Euler-Lagrange Equations

- A general set of differential equation that describe the time evolution of mechanical systems subject to holonomic constraints
- Two distinct ways of deriving these equations
 - Virtual work
 - Hamilton's principle

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Motivation

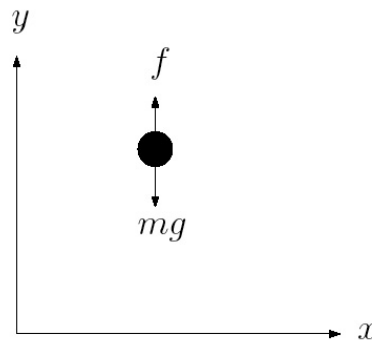


Figure 7.1: A particle of mass m constrained to move vertically constitutes a one-degree-of-freedom system. The gravitational force mg acts downward and an external force f acts upward.

$$m\ddot{y} = f - mg$$

$$m\ddot{y} = \frac{d}{dt}(m\dot{y}) = \frac{d}{dt} \frac{\partial}{\partial \dot{y}} \left(\frac{1}{2} m \dot{y}^2 \right) = \frac{d}{dt} \frac{\partial K}{\partial \dot{y}} \quad mg = \frac{\partial}{\partial y} (mgy) = \frac{\partial P}{\partial y}$$

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Motivation

$$L = K - P = \frac{1}{2} m \dot{y}^2 - mgy \quad \text{Lagrangian}$$

$$\frac{\partial L}{\partial \dot{y}} = \frac{\partial K}{\partial \dot{y}} \quad \text{and} \quad \frac{\partial L}{\partial y} = -\frac{\partial P}{\partial y}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = f \quad \text{Euler-Lagrange Equation}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = \tau_k; \quad k = 1, \dots, n \quad (q_1, \dots, q_n)$$

Generalized coordinates

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Example 7.1: Single link manipulator

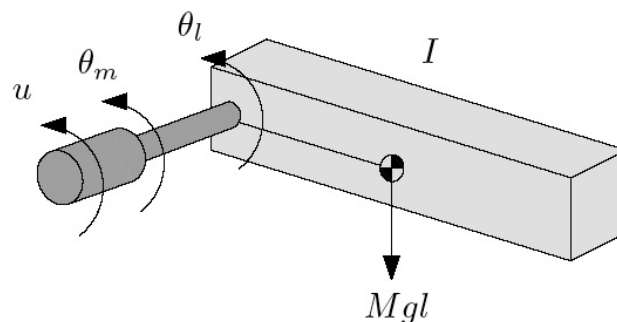


Figure 7.2: Single-link robot. The motor shaft is coupled to the axis of rotation of the link through a gear train which amplifies the motor torque and reduces the motor speed.

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Kinetic Energy of a Rigid Object

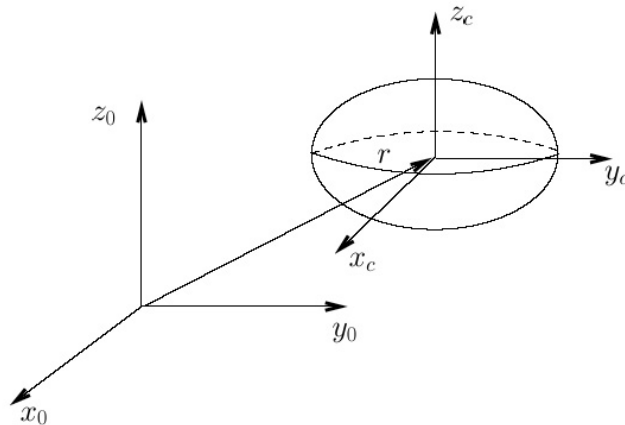


Figure 7.5: A general rigid body has six degrees of freedom. The kinetic energy consists of kinetic energy of rotation and kinetic energy of translation.

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Example 7.2: Uniform Rectangular Solid

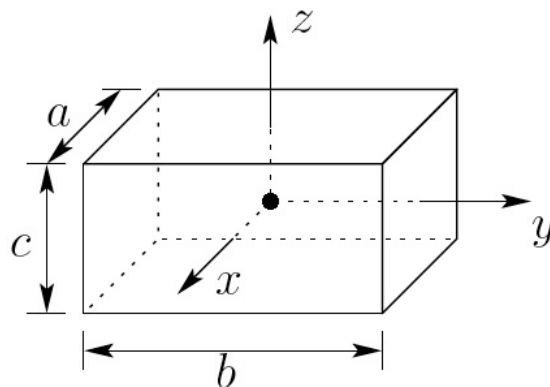


Figure 7.6: A rectangular solid with uniform mass density and coordinate frame attached at the geometric center of the solid.

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Two-links Cartesian Manipulators

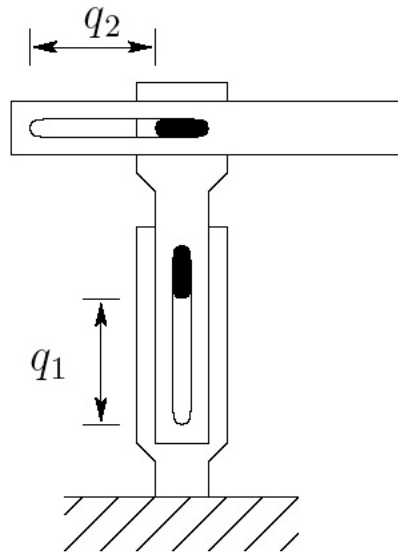


Figure 7.7: Two-link planar Cartesian robot. The orthogonal joint axes and linear joint motion of the Cartesian robot result in simple kinematics and dynamics.

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Planar Elbow Manipulator

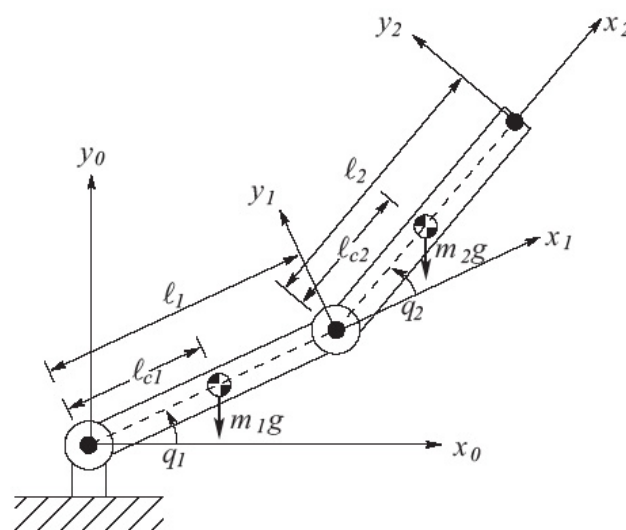


Figure 7.8: Two-link revolute joint arm. The rotational joint motion introduces dynamic coupling between the joints.

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Planar Elbow Manipulator with Remotely Driven Link

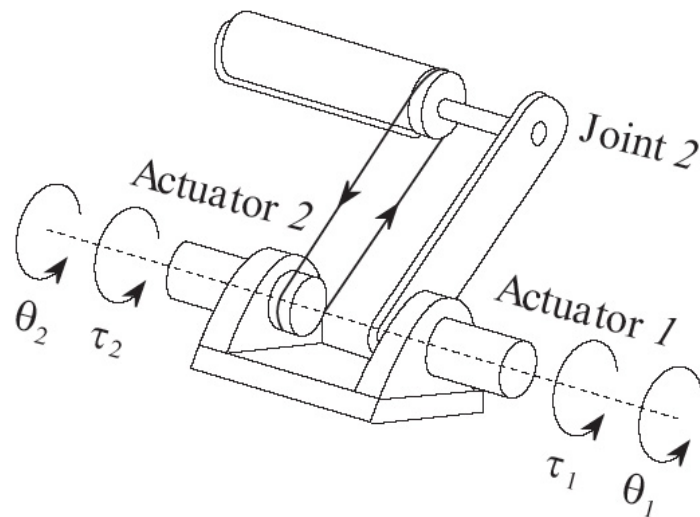


Figure 7.9: Two-link revolute joint arm with remotely driven link. Because of the remote drive the motor shaft angles are not proportional to the joint angles.

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Planar Elbow Manipulator with Remotely Driven Link

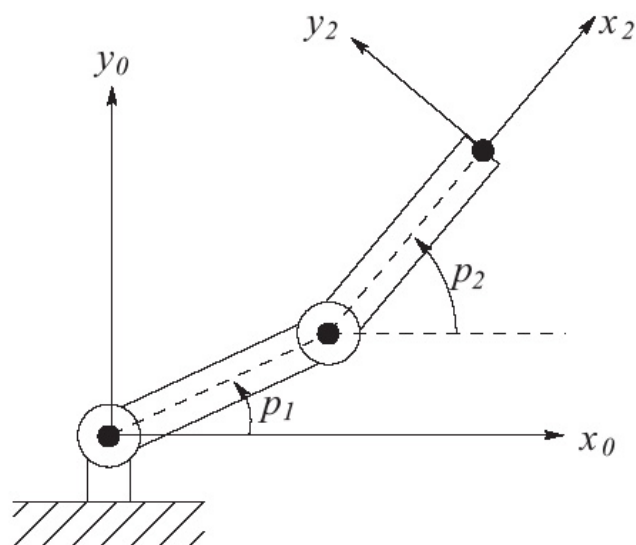


Figure 7.10: Generalized coordinates for the robot of Figure 6.4.

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Five-Bar Linkage

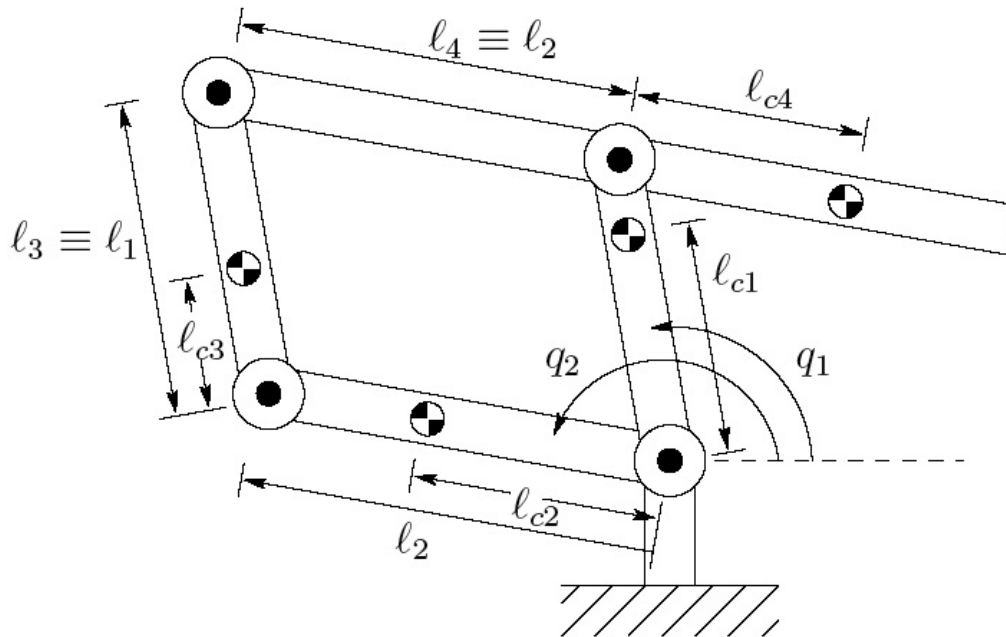


Figure 7.11: Five-bar linkage.

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Properties of Robot Dynamic Equations

- Fortunately, robot dynamic equations contain some important structural properties
 - Skew symmetry property
 - Passivity property
 - Linearity-in-the-parameters property
 - Global bounds of inertia matrix for revolute joint robots
- Good advantage for developing control algorithms

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