Binary Systems

Jee-Hwan Ryu

School of Mechanical Engineering
Korea University of Technology and Education

Binary Numbers

- $a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$ base or radix

  $= a_nr^n+a_{n-1}r^{n-1}+\ldots+a_2r^2+a_1r+a_0+a_{-1}r^{-1}+a_{-2}r^{-2}+\ldots+a_{-m}r^{-m}$

  $7392 = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$

  $(11010.11)_2 = 1\times2^4+1\times2^3+0\times2^2+1\times2^1+0\times2^0+1\times2^{-1}+1\times2^{-2}$

  $= (26.75)_{10}$

2^{10} = 1 Kilo

2^{20} = 1 Mega

2^{30} = 1 Giga

2^{40} = 1 Tera

Table 1-1
Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Binary Number

Augend	101101	Minuend:	101101	Multiplicand:	101
Addend	+100111	Subtrahend:	−100111	Multiplier:	*101
sum	1010100	Difference:	000110		101
					000
					101
				Product:	11001

Number Base Conversions

- Ex 1−1) Convert decimal 41 to binary.

	Integer Quotient		Remainder	Coefficient
41/2 =	20	+	½	$a_0 = 1$
20/2 =	10	+	0	$a_1 = 0$
10/2 =	5	+	0	$a_2 = 0$
5/2 =	2	+	½	$a_3 = 1$
2/2 =	1	+	0	$a_4 = 0$
1/2 =	0	+	½	$a_5 = 1$

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

answer : $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$

Number Base Conversions

- Ex 1-2) Convert decimal 153 to octal.

153	
19	1
2	3
0	2

$= (231)_8$

- Ex 1-3) Convert $(0.6875)_{10}$ to binary.

	Integer		Fraction	Coefficient
0.6875*2 =	1	+	0.3750	$a_{-1} = 1$
0.3750*2 =	0	+	0.7500	$a_{-2} = 0$
0.7500*2 =	1	+	0.5000	$a_{-3} = 1$
0.5000*2 =	1	+	0.0000	$a_{-4} = 1$

Answer: $(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$

Number Base Conversions

- Ex 1-4) Convert $(0.513)_{10}$ to octal.

		Coefficient
0.513 *8 =	4.104	$a_{-1} = 4$
0.104 *8 =	0.832	$a_{-2} = 0$
0.832 *8 =	6.656	$a_{-3} = 6$
0.656 *8 =	5.248	$a_{-4} = 5$
0.248 * 8 =	1.984	$a_{-5} = 1$
0.984 * 8 =	7.872	$a_{-6} = 7$

Answer: $(0.513)10 = (0.a_{-1}a_{-2}a_{-3}a_{-4....})_2 = (0.406517.....)_8$

Octal and Hexadecimal Numbers

Table 1-2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$(\underline{10}\ \underline{110}\ \underline{001}\ \underline{101}\ \underline{011}\ .\ \underline{111}\ \underline{100}\ \underline{000}\ \underline{110}\ )_2 = (26153.7460)_8$

2 6 1 5 3 7 4 0 6

$(\underline{10}\ \underline{1100}\ \underline{0110}\ \underline{1011}\ .\ \underline{1111}\ \underline{0010}\ )_2 = (2C6B.F2)_{16}$

2 C 6 B F 2

Examples

1. Convert decimal 34.4375 to binary

2. Convert the following numbers with the indicated bases to decimal: (4310)5 and (198)12

3. Convert the hexadecimal number 68BE to binary and then from binary convert it to octal

Complements

- Complements are used in digital computers for simplifying the subtraction operation and for logical manipulation.
- Two types of complements for each base-r system
  - radix complement or r's complement
  - Diminished radix complement or r-1's complement
- For binary numbers, r = 2, 2's complement or 1's complement
- For decimal numbers, r = 10, 10's complement or 9's complement

Complements

- (r-1)'s complements of N is $(r^n-1)-N$
- r=10, r-1=9, 9'complements of N is $(10^n-1)-N$
  Ex) the 9's complements of 546700 is 999999-546700 = 453299
  the 9's complements of 012398 is 999999-012398 = 987601
- For binary number, r=2, r-1=1
  1'complements of N is $(2^n-1)-N$

  Ex) the 1's complements of 1011000 is 0100111
  the 1's complements of 0101101 is 1010010

Complements

- The r's complements of an n-digit number N is

  r^n−N, N≠0

  0,   N=0

- r^n−N=[(r^n−1)−N]+1

  => The r's complements is obtained by adding 1 to the (r−1)'s complements

- Ex) The 10's complements of 012398 is 987602

  The 10's complements of 246700 is 753300

  The 2's complements of 1101100 is 0010100

  The 2's complements of 0110111 is 1001001

Examples

1. Find the 9's and 10's-complement of the following decimal numbers:

   (a) 98127634   (b) 72049900   (c) 10000000

   (d) 00000000

2. Find the 16's- complement of AF3B

3. Obtain the 1's and 2's complements of the following binary numbers:

   (a) 11101010   (b) 01111110   (c) 00000001

Complements – Subtraction with Complements

- Ex1-5) using 10's complement, subtract 72532-3250.

M =	72532
10's complement of N =	+ 96750
Sum =	169282
Discard end carry 10^5 =	−100000
Answer =	69282

- Ex1-6) Using 10's complement, subtract 3250-72532.

M =	03250
10's complement of N =	+27468
Sum =	30718

There is no end carry
Therefore, the answer is –(10's complement of 30718)=-69282

Complements – Subtraction with Complements

- Ex1-7) X=1010100, Y=1000011, (a) X-Y, (b) Y-X

(a) X-Y

X =	1010100
2's complement of Y =	+0111101
Sum =	10010001
Discard end carry 2^7 =	−10000000
Answer: X-Y =	0010001

(b) Y-X

Y =	1000011
2's complement of X =	+0101100
Sum =	1101111

There is no carry.

The answer is Y-X = -(2's complement of 1101111)=-0010001

Complements – Subtraction with Complements

- Ex1-8) Repeat Example 1-7 using 1'complement.

(a) X-Y = 1010100-1000011

X =	1010100
1's complement of Y =	+0111100
Sum =	10010000
End-around carry =	+ 1
Answer: X-Y =	0010001

(b) Y-X = 10000011-1010100

Y =	1000011
1's complement of X =	+0101011
Sum =	1101110

There is no carry.

The answer is Y-X = -(1's complement of 1101110)=-0010001

Complement of a number N contains radix point

- EX.) Complement of 10011.101001 and 732.548

Signed Binary Numbers

- Ex) The number 9 represented in binary with eight bit

  +9 : 00001001

  −9 : 10001001 (signed−magnitude representation)

  11110110 (signed−1's−complement representation)

  11110111 (signed−2's−complement representation)

Table 1-3
Signed Binary Numbers

Decimal	Signed-2's complement	Signed-1's complement	Signed magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Signed Binary Numbers

- Arithmetic Addition

– signed−magnitude system follows the rules of ordinary arithmetic.

– signed−complement system requires only addition.

2's complement

+6	00000110	−6	11111010
+13	00001101	+13	00001101
+19	00010011	+7	00000111
+6	00000110	−6	11111010
−13	11110011	−13	11110011
−7	11111001	−19	11101101

If the sum obtained after the addition is negative, it is 2's−complement form

Problems of Signed-magnitude System

- (+25) + (−37) = −(37 – 25) = −12

1. If the signs are the same, → sign comparison operation
   - add the two magnitudes → addition operation
   - give the sum the common sign

2. If the signs are different, → sign comparison operation
   - subtract the smaller magnitude from the larger → subtraction
   - give the result the sign of the larger magnitude

→ sign과 magnitude 비교, addition or subtraction 필요

Signed Binary Numbers

If 8 bit signed-complement system

+64	01000000	−64	11000000
+65	01000001	−65	10111111
+129	10000001 (?)	−129	01111111 (?)

8-bit signed-complement system의 Range: −128 ~ +127

Overflow

- If two n-bit numbers, the sum occupies n+1 bits, we say that an overflow occurs.
- Overflow is a problem in computers.

Arithmetic Subtraction

1. Take the 2's complement of the subtrahend (including the sign bit)
2. Add it to the minuend(including the sign bit)
3. A carry out of the sign-bit position is discarded

- Arithmetic Subtraction

$$(\pm A)-(+B) = (\pm A)+(-B)$$
$$(\pm A)-(-B) = (\pm A)+(+B)$$

-> A subtraction operation can be changed to an addition operation if the sign of the subtrahend is changed.

- Computers need only one common hardware circuit to handle both types of arithmetic.

Binary Code-BCD code(Binary Code for Decimal)

- the 4-bit code for one decimal

$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$

- BCD Addition

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

Table 1-4
Binary Coded Decimal (BCD)

Decimal symbol	BCD digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- if the binary sum is greater or equal to 1010, we add 0110 to obtain the correct BCD (This is because the difference between msb of the sum and a decimal carry differ by 16-10 = 6.)

Binary Code–Other Decimal Codes

Table 1-5
Four Different Binary Codes for the Decimal Digits

Decimal digit	BCD 8421	2421	Excess-3	8 4-2-1
0	0000	0000	0011	0 0 0 0
1	0001	0001	0100	0 1 1 1
2	0010	0010	0101	0 1 1 0
3	0011	0011	0110	0 1 0 1
4	0100	0100	0111	0 1 0 0
5	0101	1011	1000	1 0 1 1
6	0110	1100	1001	1 0 1 0
7	0111	1101	1010	1 0 0 1
8	1000	1110	1011	1 0 0 0
9	1001	1111	1100	1 1 1 1
	1010	0101	0000	0 0 0 1
Unused	1011	0110	0001	0 0 1 0
bit	1100	0111	0010	0 0 1 1
combi-	1101	1000	1101	1 1 0 0
nations	1110	1001	1110	1 1 0 1
	1111	1010	1111	1 1 1 0

Conversion Flow Chart

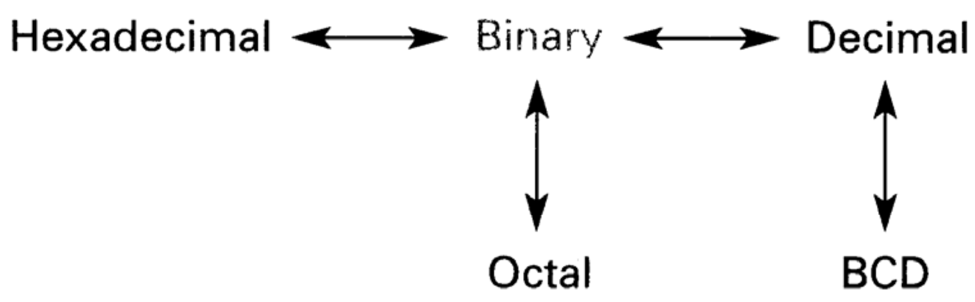


FIGURE 1-2 Conversion flowchart

Binary Code–Gray Code

Table 1-6
Gray Code

Gray code	Decimal equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

Gray Code의 장점
과 용도는?

Binary Code– ASCII Character Code

Table 1-7
American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	‘	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	−	=	M	]	m	}
1110	SO	RS	.	>	N	∧	n	~
1111	SI	US	/	?	O	−	o	DEL

Binary Code–Error-Detecting Code

- Error-Detecting Code: To detect errors in data communication and processing, and eighth bit is sometimes added to the ASCII character.

	With even parity	With odd parity
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

Examples

1. Decode the following ASCII code: 1001010 1100001 1101110 1100101 01000000 1000100 1101111 1100101

   Answer: Jane Doe

2. Represent decimal number 6027 in (a) BCD, (b) excess-3 code, (c) 2421 code

   Answer: a) 0110 0000 0010 0111

   b) Excess-3: 1001 0011 0101 1010

   c) 2421: 1100 0000 0010 1101

Binary Storage and Registers

- Registers – A register with n cells can store any discrete quantity of information that contains n bits.
- Register Transfer

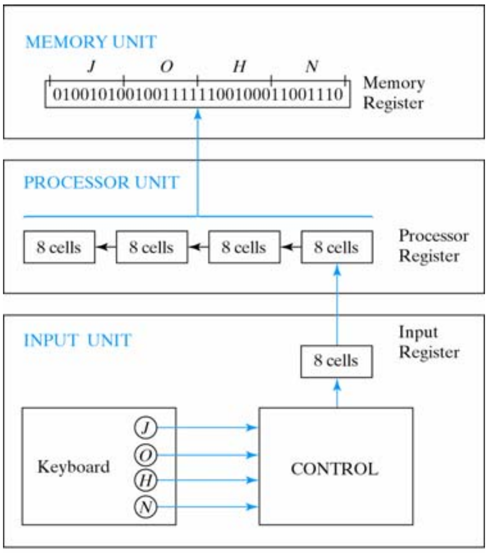


Fig. 1-1 Transfer of information with registers

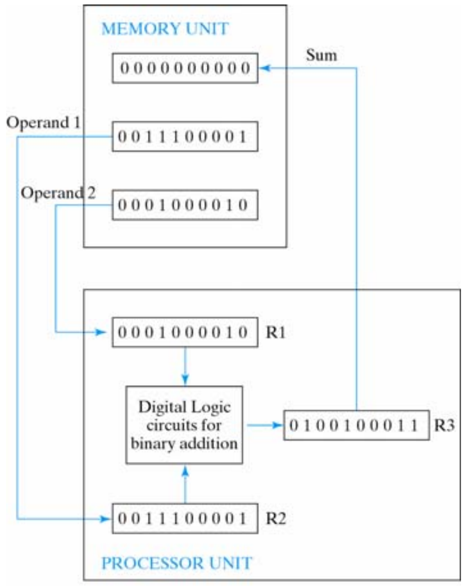


Fig. 1-2 Example of binary information processing

Binary Logic

- Definition of Binary Logic

Table 1-8
Truth Tables of Logical Operations

AND			OR			NOT	
x	y	$x \cdot y$	x	y	$x + y$	x	x'
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

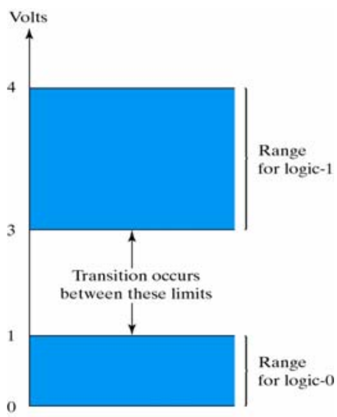
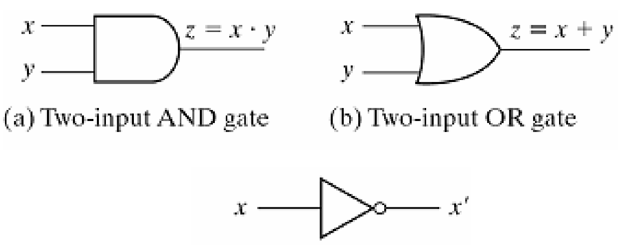


Fig. 1-3 Example of binary signals

- Logic Gates



(a) Two-input AND gate (b) Two-input OR gate

(c) NOT gate or inverter

Fig. 1-4 Symbols for digital logic circuits

Korea University of Technology and Education

Binary Logic

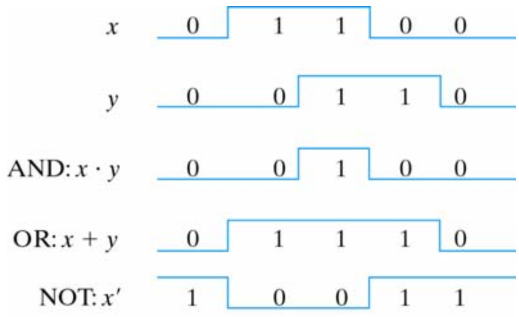
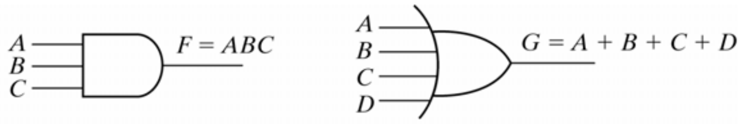


Fig. 1-5 Input-output signals for gates



(a) Three-input AND gate (b) Four-input OR gate

Fig. 1-6 Gates with multiple inputs