

Binary Systems

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Binary Numbers

- $$a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$$

$$= a_n r^n + a_{n-1} r^{n-1} + \dots + a_2 r^2 + a_1 r + a_0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \dots + a_{-m} r^{-m}$$

base or radix

$$7392 = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

$$(11010.11)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$= (26.75)_{10}$$

Table 1-1
Powers of Two

	n		n		n	
$2^{10} = 1$ Kilo	0	1	8	256	16	65,536
$2^{20} = 1$ Mega	1	2	9	512	17	131,072
$2^{30} = 1$ Giga	2	4	10	1,024	18	262,144
$2^{40} = 1$ Tera	3	8	11	2,048	19	524,288
	4	16	12	4,096	20	1,048,576
	5	32	13	8,192	21	2,097,152
	6	64	14	16,384	22	4,194,304
	7	128	15	32,768	23	8,388,608

Binary Number

Augend	101101	Minuend:	101101	Multiplicand:	101
Addend	+100111	Subtrahend:	-100111	Multiplier:	*101
sum	<u>1010100</u>	Difference:	<u>000110</u>		<u>101</u>
					000
					101
				Product:	<u>11001</u>

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Number Base Conversions

- Ex 1-1) Convert decimal 41 to binary.

	Integer Quotient		Remainder	Coefficient	Integer	Remainder
					41	
41/2 =	20	+	½	$a_0 = 1$	20	1
20/2 =	10	+	0	$a_1 = 0$	10	0
10/2 =	5	+	0	$a_2 = 0$	5	0
5/2 =	2	+	½	$a_3 = 1$	2	0
2/2 =	1	+	0	$a_4 = 0$	1	1
1/2 =	0	+	½	$a_5 = 1$	0	1

answer : $(41)_{10} = (a_5 a_4 a_3 a_2 a_1 a_0)_2 = (101001)_2$

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Number Base Conversions

- Ex 1-2) Convert decimal 153 to octal.

$$\begin{array}{r|l}
 153 & \\
 19 & 1 \\
 2 & 3 \\
 0 & 2 \quad = (231)_8
 \end{array}$$

- Ex 1-3) Convert $(0.6875)_{10}$ to binary.

	Integer		Fraction	Coefficient
$0.6875 \cdot 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \cdot 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \cdot 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \cdot 2 =$	1	+	0.0000	$a_{-4} = 1$

$$\text{Answer: } (0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$$

Number Base Conversions

- Ex 1-4) Convert $(0.513)_{10}$ to octal.

		Coefficient
$0.513 \cdot 8 =$	4.104	$a_{-1} = 4$
$0.104 \cdot 8 =$	0.832	$a_{-2} = 0$
$0.832 \cdot 8 =$	6.656	$a_{-3} = 6$
$0.656 \cdot 8 =$	5.248	$a_{-4} = 5$
$0.248 \cdot 8 =$	1.984	$a_{-5} = 1$
$0.984 \cdot 8 =$	7.872	$a_{-6} = 7$

$$\text{Answer: } (0.513)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}\dots)_8 = (0.406517\dots)_8$$

Octal and Hexadecimal Numbers

Table 1-2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$(\underline{10} \ \underline{110} \ \underline{001} \ \underline{101} \ \underline{011} \ . \ \underline{111} \ \underline{100} \ \underline{000} \ \underline{110})_2 = (26153.7460)_8$$

2 6 1 5 3 7 4 0 6

$$(\underline{10} \ \underline{1100} \ \underline{0110} \ \underline{1011} \ . \ \underline{1111} \ \underline{0010})_2 = (2C6B.F2)_{16}$$

2 C 6 B F 2

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Examples

1. Convert decimal 34.4375 to binary
2. Convert the following numbers with the indicated bases to decimal: $(4310)_5$ and $(198)_{12}$
3. Convert the hexadecimal number 68BE to binary and then from binary convert it to octal

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Complements

- ❑ Complements are used in digital computers for simplifying the subtraction operation and for logical manipulation.
- ❑ Two types of complements for each base- r system
 - radix complement or r 's complement
 - Diminished radix complement or $r-1$'s complement
- ❑ For binary numbers, $r = 2$, 2's complement or 1's complement
- ❑ For decimal numbers, $r = 10$, 10's complement or 9's complement

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Complements

- ❑ $(r-1)$'s complements of N is $(r^n-1)-N$
- ❑ $r=10$, $r-1=9$, 9's complements of N is $(10^n-1)-N$
 - Ex) the 9's complements of 546700 is $999999-546700 = 453299$
 - the 9's complements of 012398 is $999999-012398 = 987601$
- ❑ For binary number, $r=2$, $r-1=1$
 - 1's complements of N is $(2^n-1)-N$
 - Ex) the 1's complements of 1011000 is 0100111
 - the 1's complements of 0101101 is 1010010

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Complements

- The r 's complements of an n -digit number N is
$$r^n - N, N \neq 0$$
$$0, \quad N = 0$$
- $r^n - N = [(r^n - 1) - N] + 1$
 \Rightarrow The r 's complements is obtained by adding 1 to the $(r-1)$'s complements

- Ex) The 10's complements of 012398 is 987602
The 10's complements of 246700 is 753300

The 2's complements of 1101100 is 0010100
The 2's complements of 0110111 is 1001001

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Examples

1. Find the 9's and 10's-complement of the following decimal numbers:
(a) 98127634 (b) 72049900 (c) 10000000
(d) 00000000

2. Find the 16's- complement of AF3B

3. Obtain the 1's and 2's complements of the following binary numbers:
(a) 11101010 (b) 01111110 (c) 00000001

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Complements – Subtraction with Complements

- Ex1-5) using 10's complement, subtract 72532–3250.

$$\begin{array}{r}
 M = \quad 72532 \\
 10\text{'s complement of } N = \quad + 96750 \\
 \hline
 \text{Sum} = \quad 169282 \\
 \text{Discard end carry } 10^5 = \quad -100000 \\
 \hline
 \text{Answer} = \quad 69282
 \end{array}$$

- Ex1-6) Using 10's complement, subtract 3250–72532.

$$\begin{array}{r}
 M = \quad 03250 \\
 10\text{'s complement of } N = \quad +27468 \\
 \hline
 \text{Sum} = \quad 30718
 \end{array}$$

There is no end carry

Therefore, the answer is $-(10\text{'s complement of } 30718) = -69282$

Complements – Subtraction with Complements

- Ex1-7) X=1010100, Y=1000011, (a) X–Y, (b) Y–X

$$\begin{array}{r}
 \text{(a) } X-Y \\
 X = \quad 1010100 \\
 2\text{'s complement of } Y = \quad +0111101 \\
 \hline
 \text{Sum} = \quad 10010001 \\
 \text{Discard end carry } 2^7 = \quad -10000000 \\
 \hline
 \text{Answer: } X-Y = \quad 0010001
 \end{array}$$

$$\begin{array}{r}
 \text{(b) } Y-X \\
 Y = \quad 1000011 \\
 2\text{'s complement of } X = \quad +0101100 \\
 \hline
 \text{Sum} = \quad 1101111
 \end{array}$$

There is no carry.

The answer is $Y-X = -(2\text{'s complement of } 1101111) = -0010001$

Complements – Subtraction with Complements

- Ex1-8) Repeat Example 1-7 using 1's complement.

$$\begin{array}{r} \text{(a) } X - Y = 1010100 - 1000011 \\ X = \quad \quad \quad 1010100 \\ \text{1's complement of } Y = \quad \quad +0111100 \\ \hline \text{Sum} = \quad \quad \quad 10010000 \\ \text{End-around carry} = \quad \quad + \quad \quad 1 \\ \hline \text{Answer: } X - Y = \quad \quad \quad 0010001 \end{array}$$

$$\begin{array}{r} \text{(b) } Y - X = 1000011 - 1010100 \\ Y = \quad \quad \quad 1000011 \\ \text{1's complement of } X = \quad \quad +0101011 \\ \hline \text{Sum} = \quad \quad \quad 1101110 \end{array}$$

There is no carry.

The answer is $Y - X = -(1\text{'s complement of } 1101110) = -0010001$

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Complement of a number N contains radix point

- EX.) Complement of 10011.101001 and 732.548

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Signed Binary Numbers

- Ex) The number 9 represented in binary with eight bit

+9 : 00001001

-9 : 10001001 (signed-magnitude representation)

11110110 (signed-1's-complement representation)

11110111 (signed-2's-complement representation)

Table 1-3
Signed Binary Numbers

Decimal	Signed-2's complement	Signed-1's complement	Signed magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	—	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
-8	1000	—	—

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Signed Binary Numbers

- Arithmetic Addition
 - signed-magnitude system follows the rules of ordinary arithmetic.
 - signed-complement system requires **only addition**.

2's complement	+6	00000110	-6	11111010
	+13	00001101	+13	00001101
	<hr/>			
	+19	00010011	+7	00000111
2's complement	+6	00000110	-6	11111010
	-13	11110011	-13	11110011
	<hr/>			
	-7	11111001	-19	11101101

If the sum obtained after the addition is negative, it is 2's-complement form

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Problems of Signed-magnitude System

• $(+25) + (-37) = -(37 - 25) = -12$

1. If the signs are the same, → **sign comparison** operation
 - add the two magnitudes → **addition** operation
 - give the sum the common sign
2. If the signs are different, → **sign comparison** operation
 - subtract the smaller magnitude from the larger → **subtraction**
 - give the result the sign of the larger magnitude

→ **sign과 magnitude 비교, addition or subtraction 필요**

Signed Binary Numbers

If 8 bit signed-complement system

+64	01000000	-64	11000000
+65	01000001	-65	10111111
<hr/>	<hr/>	<hr/>	<hr/>
+129	10000001 (?)	-129	01111111 (?)

8-bit signed-complement system의 Range: $-128 \sim +127$

Overflow

- If two n-bit numbers, the sum occupies n+1 bits, we say that an overflow occurs.
- Overflow is a problem in computers.

Arithmetic Subtraction

1. Take the 2's complement of the subtrahend (including the sign bit)
2. Add it to the minuend(including the sign bit)
3. A carry out of the sign-bit position is discarded

- Arithmetic Subtraction

$$(\pm A) - (+B) = (\pm A) + (-B)$$

$$(\pm A) - (-B) = (\pm A) + (+B)$$

-> A subtraction operation can be changed to an addition operation if the sign of the subtrahend is changed.

- Computers need only one common hardware circuit to handle both types of arithmetic.

Binary Code-BCD code(Binary Code for Decimal)

- the 4-bit code for one decimal

$$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$$

- BCD Addition

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

Table 1-4
Binary Coded Decimal (BCD)

Decimal symbol	BCD digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- if the binary sum is greater or equal to 1010, we add 0110 to obtain the correct BCD (This is because the difference between msb of the sum and a decimal carry differ by $16-10 = 6$.)

Binary Code—Other Decimal Codes

Table 1-5
Four Different Binary Codes for the Decimal Digits

Decimal digit	BCD 8421	2421	Excess-3	8 4-2-1
0	0000	0000	0011	0 0 0 0
1	0001	0001	0100	0 1 1 1
2	0010	0010	0101	0 1 1 0
3	0011	0011	0110	0 1 0 1
4	0100	0100	0111	0 1 0 0
5	0101	1011	1000	1 0 1 1
6	0110	1100	1001	1 0 1 0
7	0111	1101	1010	1 0 0 1
8	1000	1110	1011	1 0 0 0
9	1001	1111	1100	1 1 1 1
	1010	0101	0000	0 0 0 1
Unused	1011	0110	0001	0 0 1 0
bit	1100	0111	0010	0 0 1 1
combi-	1101	1000	1101	1 1 0 0
nations	1110	1001	1110	1 1 0 1
	1111	1010	1111	1 1 1 0

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Conversion Flow Chart

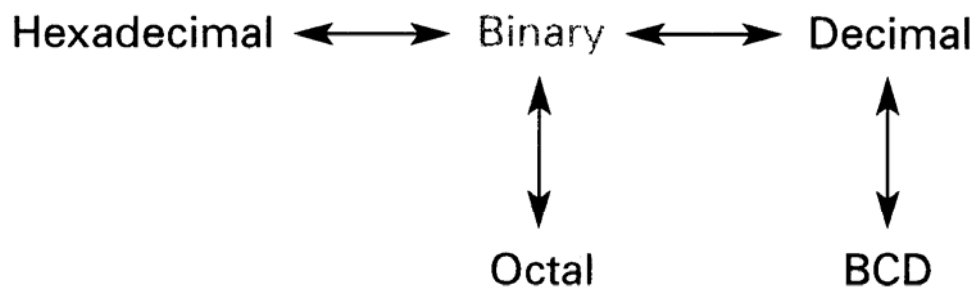


FIGURE 1-2 Conversion flowchart

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Binary Code–Gray Code

Table 1-6
Gray Code

Gray code	Decimal equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

Gray Code의 장점
과 용도는?

Binary Code– ASCII Character Code

Table 1-7
American Standard Code for Information Interchange (ASCII)

$b_4b_3b_2b_1$	$b_7b_6b_5$							
	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	\$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	·	7	G	W	g	w
1000	BS	CAN	(8	H	X	h	x
1001	HT	EM)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[k	{
1100	FF	FS	,	<	L	\	l	
1101	CR	GS	-	=	M]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	-	o	DEL

Binary Code–Error–Detecting Code

- **Error–Detecting Code:** To detect errors in data communication and processing, and eighth bit is sometimes added to the ASCII character.

	With even parity	With odd parity
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

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Examples

1. Decode the following ASCII code: 1001010 1100001 1101110
1100101 0100000 1000100 1101111 1100101

Answer: Jane Doe

2. Represent decimal number 6027 in (a) BCD, (b) excess–3 code, (c) 2421 code

Answer: a) 0110 0000 0010 0111

b) Excess–3: 1001 0011 0101 1010

c) 2421: 1100 0000 0010 1101

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Binary Storage and Registers

- Registers – A register with n cells can store any discrete quantity of information that contains n bits.
- Register Transfer

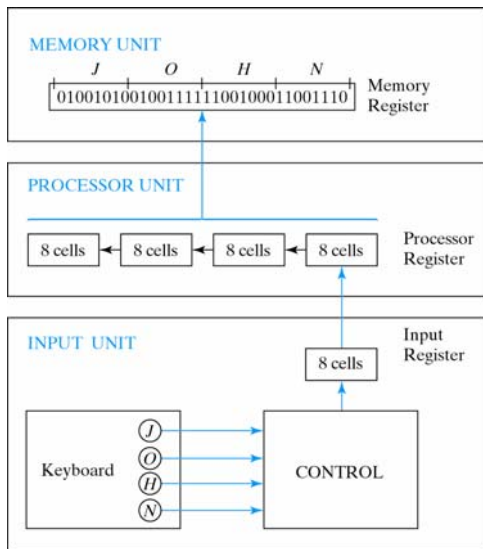


Fig. 1-1 Transfer of information with registers

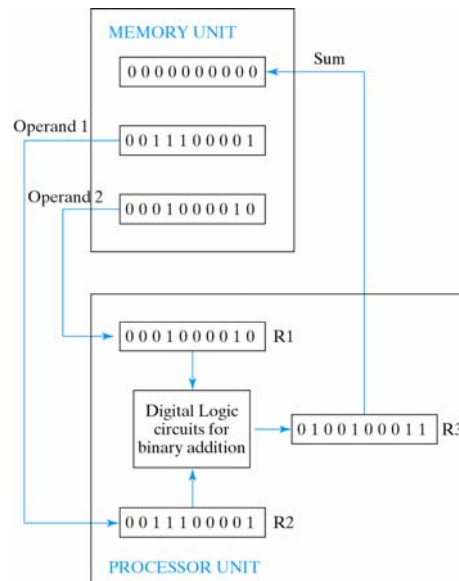


Fig. 1-2 Example of binary information processing

Binary Logic

- Definition of Binary Logic

Table 1-8
Truth Tables of Logical Operations

AND			OR			NOT	
x	y	$x \cdot y$	x	y	$x + y$	x	x'
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

- Logic Gates

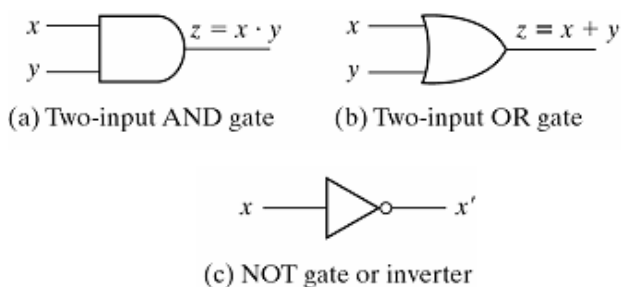


Fig. 1-4 Symbols for digital logic circuits

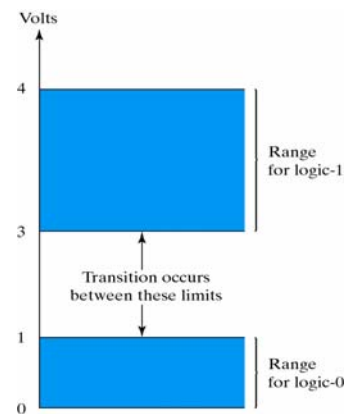


Fig. 1-3 Example of binary signals

Binary Logic

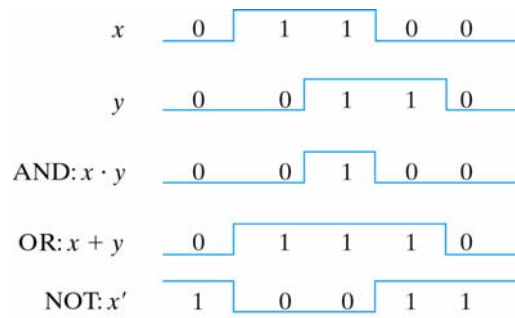


Fig. 1-5 Input-output signals for gates

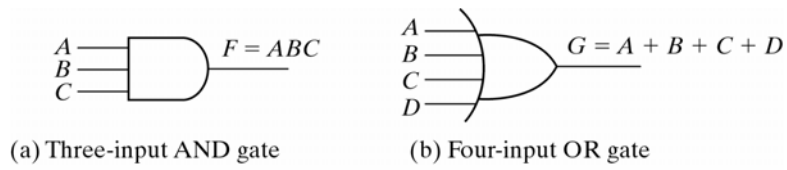


Fig. 1-6 Gates with multiple inputs