Gain and Phase Margins

Jee-Hwan Ryu

School of Mechanical Engineering
Korea University of Technology and Education

This system is stable—but how close to being unstable is it?
• This is the “stability robustness” question
• The system’s Gain Margin and Phase Margin are quantitative measures of the system’s stability robustness

Loop Transfer function:
\[ L(s) = \frac{s+5}{(s+2)s(s+1)} \]
Stability Margin

- Closed-loop transfer function is not usually known
- Would like to determine Closed-loop stability by evaluating the frequency response of open-loop transfer function $KG(jw)$
- This can be done without a math model of the system by experimentally determining the open-loop frequency response.

When pole is at imaginary axis

$$|KG(jw)| = 1 \quad \text{and} \quad \angle(KG(jw)) = -180^\circ$$

System become less stable as the gain increases

$$|KG(jw)| < 1 \quad \text{and} \quad \angle(KG(jw)) = -180^\circ$$

Bode Plot for this example
Now in discrete time....

- Replace $s$ with $z$
- Plot the bode plot (or polar plot) of the system only up to the Nyquist frequency $\pi/T$