ME2025 Digital Control

DT Signals and Dynamic Response

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Zeros and Poles of transfer functions of DT signals

- Unit Pulse

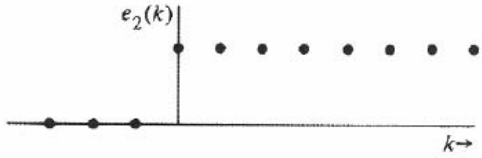
$$
\begin{aligned}
e_1(k) &= 1 \qquad (k = 0) \\
&= 0 \qquad (k \neq 0) \\
&= \delta_k;
\end{aligned}
$$

No poles, no zeros

$$E_1(z) = \sum_{-\infty}^{\infty} \delta_k \, z^{-k} = z^0 = 1$$

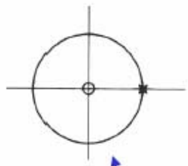
Zeros and Poles of transfer functions of DT signals

Unit Step



$$e_2(k) = 1 \qquad (k \geq 0)$$
$$= 0 \qquad (k < 0)$$
$$\widehat{=}\ 1(k) \equiv 1[\mathrm{k}] \equiv \mathrm{u}[\mathrm{k}]$$

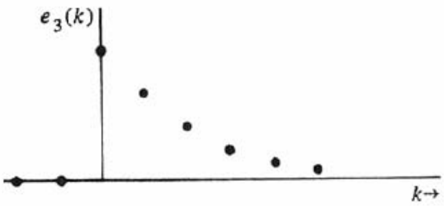
$$E_2(z) = \sum_{k=-\infty}^{\infty} e_2(k)\ z^{-k} = \sum_{k=0}^{\infty} z^{-k}$$
$$= \frac{1}{1 - z^{-1}} \qquad (|z^{-1}| < 1)$$
$$= \frac{z}{z-1} \qquad (|z| > 1).$$



1 pole, 1 zero

Zeros and Poles of transfer functions of DT signals

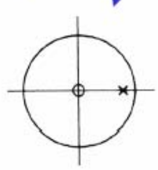
- Exponential



$$e_3(k) = r^k \qquad (k \geq 0)$$
$$= 0 \qquad (k < 0)$$
$$= r^k 1(k)$$

$$E_3(z) = \sum_{k=0}^{\infty} r^k z^{-k}$$
$$= \sum_{k=0}^{\infty} (rz^{-1})^k$$
$$= \frac{1}{1 - rz^{-1}} \qquad (|rz^{-1}| < 1)$$
$$= \frac{z}{z-r} \qquad (|z| > |r|)$$

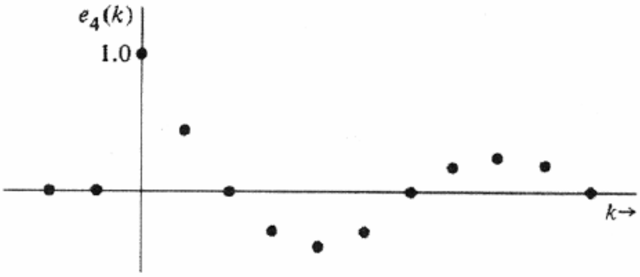
1 pole, 1 zero



Zeros and Poles of transfer functions of DT signals

- Modulated Sinusoid

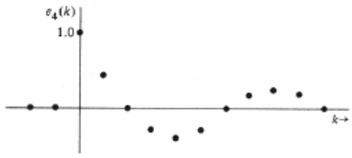
$$e_4(k) = [r^k \cos(k\theta)]1(k) \quad = r^k\left(\frac{e^{jk\theta}+e^{-jk\theta}}{2}\right)1(k)$$



Zeros and Poles of transfer functions of DT signals

- Modulated Sinusoid

$$e_4(k) = [r^k \cos(k\theta)]1(k) = r^k\left(\frac{e^{jk\theta}+e^{-jk\theta}}{2}\right)1(k)$$



Work on the first term:

$$e_5(k) = r^k e^{jk\theta} 1(k)$$

$$E_5(z) = \sum_{k=0}^{\infty} r^k e^{j\theta k} z^{-k}$$

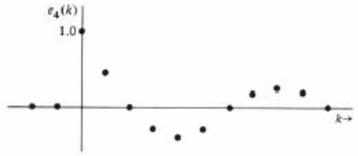
$$= \sum_{k=0}^{\infty} (re^{j\theta} z^{-1})^k$$

$$= \frac{1}{1 - re^{j\theta} z^{-1}}$$

$$= \frac{z}{z - re^{j\theta}} \qquad (|z| > r)$$

Zeros and Poles of transfer functions of DT signals

- Modulated Sinusoid



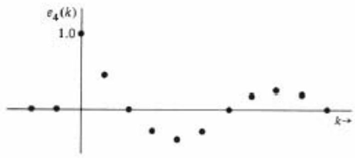
$$e_4(k) = [r^k \cos(k\theta)]1(k) \;\; = r^k \left(\frac{e^{jk\theta} + e^{-jk\theta}}{2} \right) 1(k)$$

Find second term by replacing θ by $-\theta$

$$\mathcal{Z}\{r^k e^{-j\theta k} 1(k)\} = \frac{z}{z - re^{-j\theta}} \qquad (|z| > r)$$

Zeros and Poles of transfer functions of DT signals

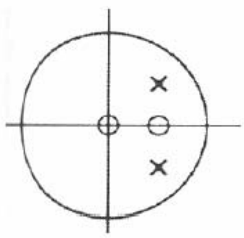
- Modulated Sinusoid



$$e_4(k) = [r^k \cos(k\theta)]1(k) \;\; = r^k \left(\frac{e^{jk\theta} + e^{-jk\theta}}{2} \right) 1(k)$$

Putting them together

$$E_4(z) = \frac{1}{2} \left\{ \frac{z}{z - re^{j\theta}} + \frac{z}{z - re^{-j\theta}} \right\}$$
$$= \frac{z(z - r\cos\theta)}{z^2 - 2r(\cos\theta)z + r^2} \qquad (|z| > r)$$



2 poles, 2 zeros

Modulated Sinusoid

- Settling time of transient mostly determined by radius of poles, r *(note: r nonnegative)*
  - If $r > 1$ time domain signal keeps growing
  - If $r = 1$, constant amplitude time domain signal
  - If $r < 1$ then signal decays
    - Smaller r settles more quickly