

Fig. 8. Actual energy behavior with the new PO/PC approach for high-stiffness VE  $K = 120$  kN·m.

## VI. CONCLUSION

In this paper, a more accurate time-domain passivity-control approach is proposed, considering the velocity change during one sample time. The actual energy output can be measured precisely with the sampled-time passivity measure, but we can only know the actual energy output after energy is already produced. To avoid the active behavior, we proposed a new PO, combining both predictive and accurate features, and designed the PC based on the new PO. We analyzed the sampled- and continuous-time energy behavior, and proved that the sampled passive system is at least stable, even though it is not passive in continuous time. The experiments showed that we could achieve stable contact with a higher virtual spring stiffness than by using the earlier technique.

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## Control of a Flexible Manipulator With Noncollocated Feedback: Time-Domain Passivity Approach

Jee-Hwan Ryu, Dong-Soo Kwon, and Blake Hannaford

**Abstract**—A new method to control a flexible manipulator with noncollocated feedback is proposed. We introduce a method to implement the time-domain passivity-control approach to a flexible manipulator with noncollocated feedback, which could not be treated with the previous time-domain passivity-control framework due to a possible active transfer function from the input to the noncollocated output. The proposed method is simulated with a single-link flexible manipulator, and a good control performance is obtained.

**Index Terms**—Flexible manipulator, noncollocated feedback, passivity controller (PC), passivity observer (PO), time-domain passivity.

## I. INTRODUCTION

Flexible manipulators are finding their way into industrial and space robotic applications due to their lighter weight and faster response time, compared with rigid manipulators. Control of flexible manipulators has been studied extensively for more than a decade by many researchers [2], [3], [12], [20], [23], [25]. Despite their results, this control problem has proven to be rather complicated.

It is well known that stabilization of a flexible manipulator can be greatly simplified by collocating the sensors and the actuator, where the input–output (I/O) mapping is passive [26], and a stable controller can be easily devised independent of the structural details. However, the performance of this collocated feedback turns out to be unsatisfactory, due to a weak control of the vibrations of the link [4]. This initiated finding other noncollocated output measurements, such a position of the end-point of the link to increase the control performance [3]. However, if the end-point is chosen as the output and the joint torque is chosen as the input, the system becomes a nonminimum phase one, and may behave actively. As a result, a small increment of feedback controller gains can easily make the closed-loop system unstable. This led many researchers to seek other outputs which have the passivity property.

Wang and Vidyasagar proposed the so-called reflected tip position as such an output [26]. This corresponds to the rigid-body deflection minus the deflection at the tip of the flexible manipulator. Pota and Vidyasagar used the same output to show that in the limit, for a nonuniform link, the transfer function from the input torque to the derivative of the reflected tip position is passive whenever the ratio of the link inertia to the hub inertia is sufficiently small [15]. Chodavarapu and Spong considered the virtual angle of rotation, which consists of the hub angle of rotation augmented with a weighted value of the slope of the link at its tip [4]. They showed that the transfer function with this output is minimum phase and that the zero dynamics are stable.

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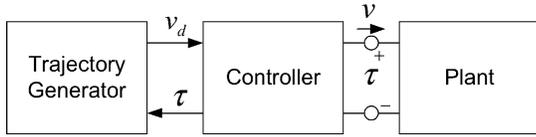


Fig. 1. Network view of a motion-control system.

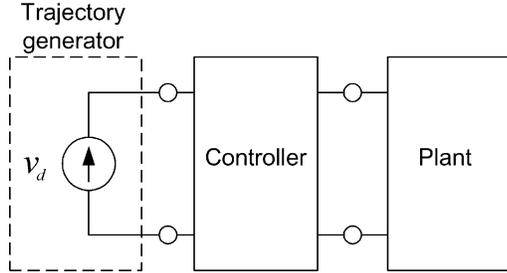


Fig. 2. Network view of general motion-control systems.

Despite the fact that these previous efforts have succeeded in numerous kinds of applications, the critical drawback is that these are model-based approaches, requiring the system parameters or the dynamic structural information at the least. However, interesting systems are uncertain, and it is usually hard to obtain the exact dynamic parameters and structural information.

To control the tip position of a flexible manipulator without system model information, several energy-based approaches have been proposed [6]–[8]. However, these approaches require additional sensory information. There are other interesting research results on simple sensor-based output feedback-control laws [13], [14].

In this paper, we introduce a different way of treating noncollocated control systems without any model information. A recently developed stability-guaranteed control method based on time-domain passivity control [9], [18], [19] is applied. First the new control method is reviewed, and then several issues for implementing noncollocated control systems are studied. Performance of the proposed controller is then investigated through numerical simulations with a flexible manipulator.

## II. REVIEW OF TIME-DOMAIN PASSIVITY APPROACH

### A. Network Model and Stability Concept

In our previous paper [19], the traditional control-system view was analyzed in terms of energy flow by representing it from a network point of view. The connection between trajectory generator and controller, which traditionally consists of a one-way command information flow, was modified by the addition of a virtual feedback of a conjugate variable. For a motion-control system, the output of the trajectory generator would be a desired velocity ( $v_d$ ), and the virtual feedback would be equal to the controller output ( $\tau$ ) (Fig. 1), and this network model was equivalently described in Fig. 2, where trajectory generator was a current (or velocity) source based on electrical-mechanical analogy.

From the circuit representation (Fig. 2), we found that the virtual input energy from the trajectory generator depends on the impedance of the connected controller and plant. It is assumed that the plant is passive. Thus, if we could make the controller (two-port network) passive, we could guarantee the stability of the system, since the passivity of each block was a sufficient condition for stability.

### B. Time-Domain Passivity Approach

In this section, we briefly review the time-domain passivity-control approach, which makes the controller (two-port network) passive. First, we define the sign convention for all forces and velocities so that their

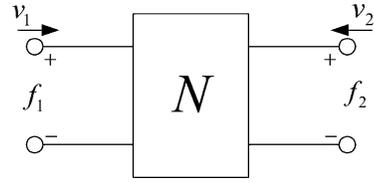


Fig. 3. Two-port network.

product is positive when power enters the system port (Fig. 3). The following well-known definition of passivity is used.

*Definition 1:* The two-port network  $N$ , with zero initial energy storage, is *passive* if and only if

$$\int_0^t f_1(\tau)v_1(\tau) + f_2(\tau)v_2(\tau)d\tau \geq 0 \quad \forall t \geq 0 \quad (1)$$

for forces ( $f_1, f_2$ ) and velocities ( $v_1, v_2$ ). Equation (1) states that the energy supplied to a passive network must be greater than zero for all time [1], [5], [22], [24].

In a computer-implemented control system, the conjugate variables that define power flow are discretized. The corresponding analysis is confined to systems having a sampling rate substantially faster than the dynamics of the system. It is assumed that there is no change in force and velocity during one sampling period. Thus, we can easily “instrument” one or more blocks in the system with the following “passivity observer” (PO) for a two-port network to check the passivity (1):

$$E_{\text{obsv}}(n) = \Delta T \sum_{k=0}^n (f_1(k)v_1(k) + f_2(k)v_2(k)) \quad (2)$$

where  $\Delta T$  is the sampling period.  $E_{\text{obsv}}(n) \geq 0$  for every  $n$  implies the system dissipates energy. If there is an instance when  $E_{\text{obsv}}(n) < 0$ , this means the system generates energy, and the amount of generated energy is  $-E_{\text{obsv}}(n)$ . Recently, other research has allowed this constant force and velocity assumption to be relaxed [17], [21].

Consider a two-port system which may be active. Depending on operating conditions and the specifics of the two-port system’s dynamics, the PO may or may not be negative at a particular time. However, if it is negative at any time, we know that the two-port may then be contributing to instability. Moreover, we know the exact amount of energy generated, and we can design a time-varying element to dissipate only the required amount of energy. We call this element a “passivity controller” (PC). The PC takes the form of a dissipative element in a series or parallel configuration, depending on the input causality [9]. Note that in the PO/PC approach, the basic assumption was that the plant which was connected to the controller two-port is passive. However, if the plant is active, the current PO/PC approach can not be applied. In the next section, we will consider the case where the plant is not passive.

Please see [9] and [17]–[19] for more detail on the time-domain passivity-control approach.

## III. IMPLEMENTATION ISSUES

This section addresses how to implement the time-domain passivity-control approach to a flexible manipulator with noncollocated feedback. Consider a single-link flexible manipulator having a planar motion, as detailed in Fig. 4.  $v_e$  is the end-point velocity,  $v_a$  is the velocity of the actuating position, and  $\tau$  is the control torque at the joint.

### A. Building the Network Model

When we feedback the end-point position to control the motion of the flexible manipulator, a network model (including causality) of the

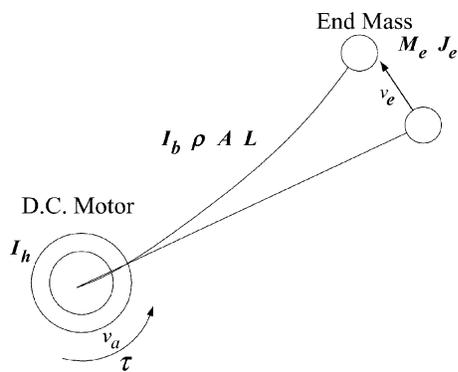


Fig. 4. Single-link flexible manipulator.

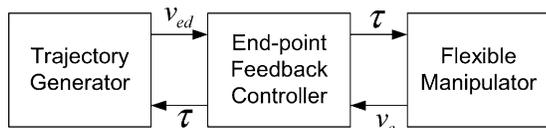


Fig. 5. Network model of flexible manipulator with end-point feedback.

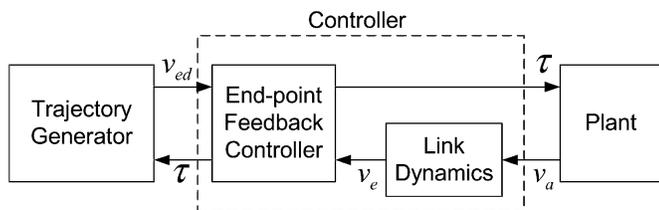


Fig. 6. Modified network model of flexible manipulator with end-point feedback.

overall control system can be derived (Fig. 5).  $v_{ed}$  means a desired velocity of the end-point. In this case, we have to consider one important thing. If the I/O relation of the plant is active, the time-domain passivity-control scheme cannot be applied, since the time-domain passivity-control scheme has been developed with the framework that the I/O relation of the plant is passive. If the end-point position is a plant output and joint torque is a plant input, the I/O relation of the plant is possibly active. Thus, the overall control system may not be passive, even though the controller remains passive.

To solve this problem, we change the above network model to be more suitable to our framework. The important physical fact is that the conjugate I/O pair  $(v_e, \tau)$  is not simulating physical input energy from the controller to the flexible manipulator. The physical energy only flows into the flexible manipulator through the place where the actuator is attached. Even though the controller uses noncollocated sensor information to generate its output, the actual physical energy that is transmitted to the flexible manipulator is determined by the conjugate pair at the actuating position. Therefore, we can extract link dynamics, from the joint velocity to the end-point velocity, from the flexible manipulator (which has noncollocated feedback), and include it in the controller block (Fig. 6). The noncollocated (possibly active) system is then separated into the collocated (passive) system and a dynamics from the collocated output (joint velocity) to the noncollocated output (end-point velocity). As a result, if it is possible, and it generally is, to use the velocity information measured at the actuating position, we can construct the network model (controller and passive plant) that is suitable to our framework, as in Fig. 6, by including the link dynamics that cause the noncollocation problem into the controller.

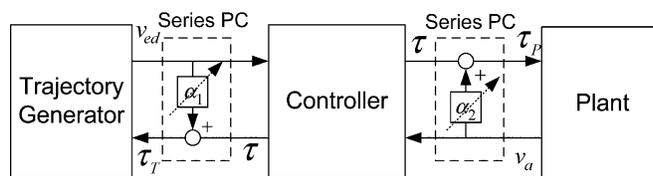


Fig. 7. Configuration of PC for a flexible manipulator with end-point feedback.

### B. Designing the PO/PC

First, for designing the PO, it is necessary to check the real-time availability of the conjugate signal pairs at each port of the controller. The conjugate pair at the port that is connected with the trajectory generator is usually available, since the desired trajectory ( $v_{ed}$ ) is given, and the controller output ( $\tau$ ) is calculated in real time. Furthermore, the conjugate pair is generally available for the other port that is connected to the plant, since the same controller output is used, and the output velocity of the actuating position ( $v_a$ ) is measured in real time. Thus, the PO is designed as

$$E_{\text{obsv}}(n) = \Delta T \sum_{k=0}^n (\tau(k)v_{ed}(k) - \tau(k)v_a(k)). \quad (3)$$

After designing the PO, the causality of each port of the controller should be determined in order to choose the type of PC for implementation. In a noncollocated flexible manipulator control system, the output of the trajectory generator is the desired velocity ( $v_{ed}$ ) of the end-point, and the controller output ( $\tau$ ) is feedback to the trajectory generator. Thus, the port that is connected with the trajectory generator has impedance causality. Also, the other port of the controller has impedance causality, because a motion-controlled flexible manipulator usually has admittance causality (torque input  $\tau$  and joint velocity output  $v_a$ ). Thus, two series PCs have to be placed at each port of the controller (Fig. 7)

$$\tau_T = \tau + \alpha_1 v_{ed}, \quad \tau_P = \tau + \alpha_2 v_a \quad (4)$$

where  $\tau_P$  and  $\tau_T$  are modified controller outputs to the trajectory generator and plant, respectively.

Please see [18] for a more detailed PC algorithm which is used in this system.

## IV. SIMULATION EXAMPLES

Many researchers have used a flexible manipulator for testing newly developed control methods, due to its significant control challenges. In this section, the proposed stability-guaranteed control scheme for noncollocated control systems is tested for feasibility with a simulated flexible-link manipulator.

The experimentally verified single-link flexible manipulator model [11] is employed in this paper. A single-link flexible manipulator having a planar motion is detailed in Fig. 4. The rotational inertia of the servo motor, the tachometer, and the clamping hub are modeled as a single-hub inertia  $I_h$ . The payload is modeled as an end mass  $M_e$  and a rotational inertia  $J_e$ . The joint friction is included in the damping matrix. The system parameters in Fig. 4 are given in Table I. The closed-form dynamic equation is derived using the assumed-mode method. For the system dynamic model, the flexible arm is modeled up to the third mode, that is, an eighth-order system is considered.

TABLE I  
 PHYSICAL PROPERTIES OF SINGLE-LINK FLEXIBLE MANIPULATOR

Link	Tip mass	Hub
Stiffness ( $EI$ ): $11.85Nm^2$	Mass ( $M_e$ ): $0.5867Kg$	Rotational inertia ( $I_h$ ):
Thickness ( $H$ ): $47.63e-4m$	Rotational inertia ( $J_h$ ):	$0.016Kgm^2$
Unit length mass ( $\rho A$ ): $0.2457Kg/m$	$0.2787Kgm^2$	
Length ( $L$ ): $1.1938m$		

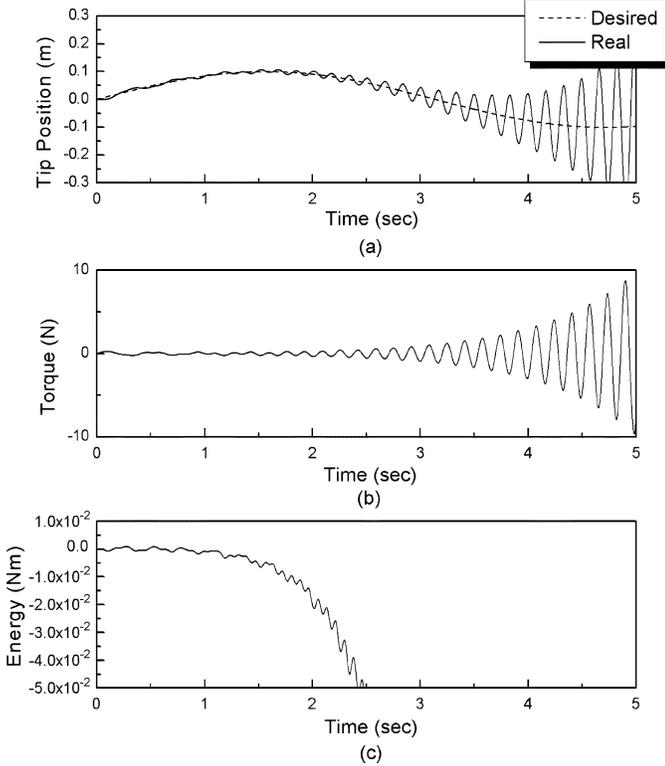


Fig. 8. Tip-position feedback without the PC.

The following noncollocated tip-position proportional-derivative (PD) controller is used:

$$\tau(k) = K_P \Delta T \sum_{j=0}^k (v_{ed}(j) - v_e(j)) + K_D (v_{ed}(k) - v_e(k)) \quad (5)$$

where  $K_P = 30$  and  $K_D = 0.8$ . In this noncollocated-feedback system, the hub angle and joint torque can be considered as a conjugate pair to calculate the physical energy output flow into the flexible manipulator (see Section III-A).

Without the PC turned on, tip-position tracking control was simulated (Fig. 8). The desired tip-position trajectory was  $x_d(t) = 0.1 \sin(t)$ . The tip position could not follow the desired trajectory, and tip position and control input have oscillation which increases with time [Fig. 8(a), (b)]. The PO [Fig. 8(c)] grew to more and more negative values.

Stable tip-position tracking is achieved with the PC turned on. Tip position tracks the desired trajectory very well [Fig. 9(a)], and the PO is constrained to positive values [Fig. 9(c)]. The PC at both sides of the controller/flexible-link block are briefly operating, only when these are required, and they dissipate only the amount of energy actually generated [Fig. 9(d)] by briefly modifying the controller output [Fig. 9(b)].

For showing the effectiveness of the proposed controller, a task with a larger desired tip motion (maximum displacement = 1.0 m) is simulated for 10 s with PC (Fig. 10). Even for the large displacement, the

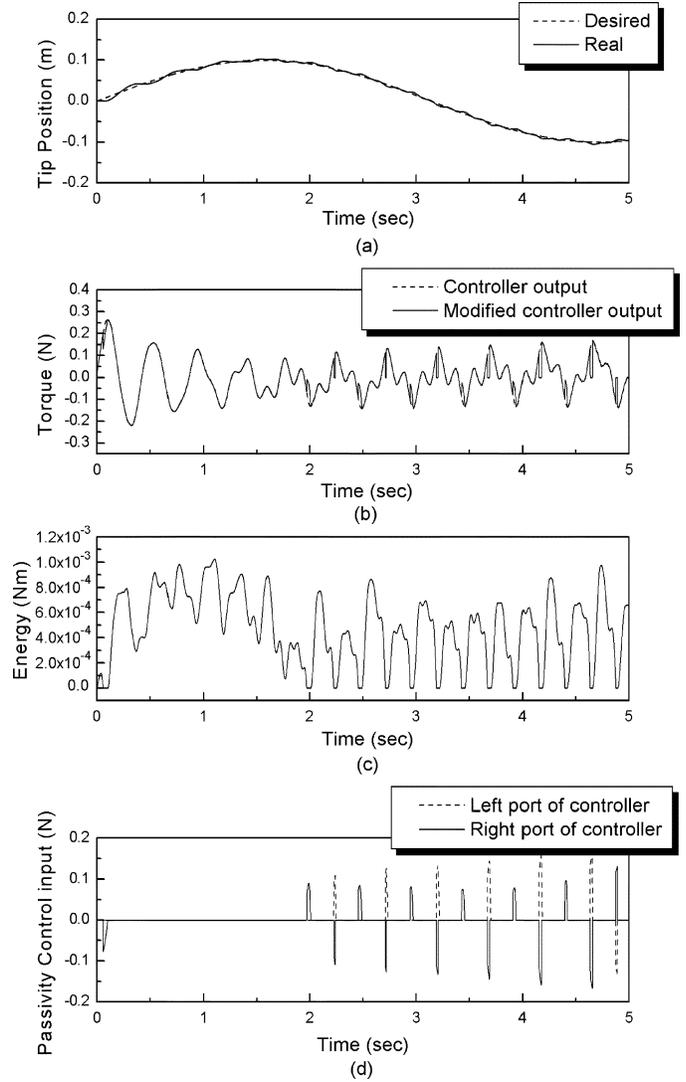


Fig. 9. Tip-position feedback with the PC.

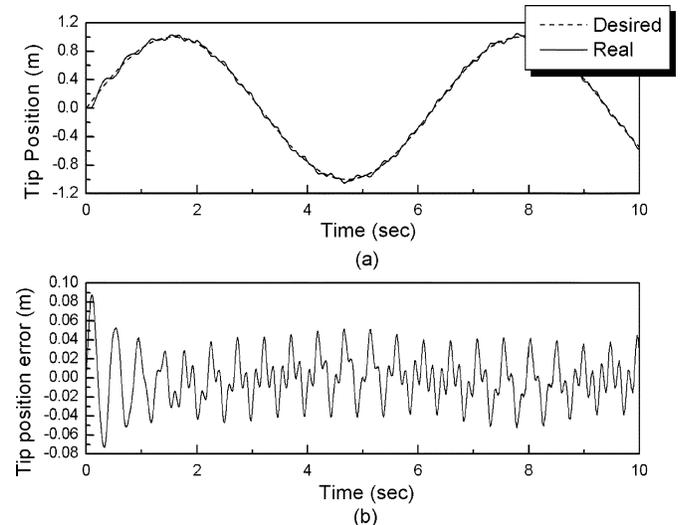


Fig. 10. Tip-position feedback with the PC for larger desired tip motion (maximum displacement = 1.0 m).

tip position tracks the desired trajectory very well [Fig. 10(a)], and the tip-position error remained less than 4% of the maximum magnitude [Fig. 10(b)].

## V. DISCUSSION

In this paper, we propose a stability-guaranteed control scheme for noncollocated feedback-control systems without any model information. The main contribution of this paper is proposing a method to implement the PO/PC for a possibly active plant, due to the noncollocated feedback. Without needing model information, we separate the active plant into a passive plant and a possibly active transfer function from the collocated output to the noncollocated output, which allows the control system to be fit to the PO/PC framework. Simulation results with an eighth-order realistic model demonstrated stable control, even for the noncollocated control system. Note that the proposed controller is independent of the order of the model. However, if the high mode of the system is excited, the PO might miss the active energy behavior of the system. We have studied this problem in [16].

As a further paper, we will prove the feasibility of the proposed PO/PC approach through an experiment with a flexible manipulator. One of the expected problems is the performance degradation, due to the noise, during low values of velocity [10], [18], [19].

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